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Evolutionary Micro-founded Technical Change and The Kaldor-Verdoorn Law: Estimates from an Artificial World*

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Abstract

This paper proposes to identify the micro-level sources for the dynamic increasing returns occurring at an aggregate level. The paper reverts to a micro model of technological change in-line with the evolutionary literature on industrial dynamics. The data generated through numerical simulations are used to identify the sources of increasing returns as measured by the Kaldor-Verdoorn Law. In this respect we also aim to provide some plausible micro-foundations to this Law. The paper shows that: (i) Dynamic increasing returns appear as an emergent property of the model; (ii) micro-characteristics of technical change, as the amplitude and the frequency of changes, as well as selection mechanisms significantly shape these increasing returns.

KEYWORDS: DYNAMIC INCREASING RETURNS, KALDOR-VERDOORN LAW, TECHNICAL CHANGE, EVOLUTIONARY MODELLING
JEL CLASSIFICATION: B52, L11, O31, O33, O40

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1 Introduction

Understanding the sources of growth and long run development of economies is an age-old issue in economics. From A. Smith's seminal work to the latest development in growth theory, economists widely acknowledge the key role played by changes in production technologies as a major source of economic growth. These technological changes are usually considered as favouring growth by preventing decreasing returns. No real consensus can, however, be found on the sources of these increasing returns.

Among the possible explanations to be found in the literature, division of labour and technological innovation are the most spread. From a formal point of view, the dominant stream of literature tends to assume the existence of increasing returns; the latter being justified by various exogenous mechanisms. Hence, the New Growth Theory (NGT) releases the assumption of an exogenous residual technical change by simply assuming increasing (or non-decreasing) returns (see Romer, 1986; Romer, 1990). Prior to the development of the NGT, some economists have tried to consider the sources of these increasing returns as resulting from the economic activity itself. Among these, Kaldor's Cumulative Causation developed, from the mid sixties onwards, a Post-Keynesian approach to economic dynamics accounting for dynamic increasing returns. Evolutionary economics constitutes a second alternative to the NGT. Drawing on Schumpeter's legacy, it accounts for technical change as the trigger of economic dynamics.

For the Kaldorians, technical change is one of the key components responsible for the cumulative causation underlying the growth process. Technical change within this framework cannot be distinguished from the existence of increasing returns. These are strongly interrelated: technical change is a source for increasing returns on the one hand. Increasing returns are a prerequisite for technical change to occur on the other. For Kaldor, the sources of increasing returns are two-fold:

First, at the firm level, increasing returns, if partly induced by large scale manufacturing activities, in line with the Smithian concept of static increasing returns, are mainly rooted in the renewal of production capacities through investments. These micro-level sources are captured in Kaldor (1957)'s 'technical progress function':

> "Hence instead of assuming that some given rate of increase in the productivity is attributable to technical progress which is superimposed, so to speak, on the growth of productivity attributable to capital accumulation, we shall postulate a single relationship between the growth of capital and the growth of productivity which incorporates

the influence of both factors.[...] The shape of [the 'technical progress function'] reflects both the magnitude and the character of technical progress as well as the increasing in organisation, etc., difficulties imposed by faster rates of technical change. It may be assumed that some increases in productivity would take place even if capital per man remained constant over time, since there are always some innovations [...] which enable production to be increased without additional investment. But beyond these the growth in productivity will depend on the rate of growth in the capital stock" (Kaldor (1957), as reprinted p 265-266 in Kaldor (1960)).

Second, increasing returns are rooted into a macro-level division of labour. The latter increases the sectoral specialisation, generates novelty and therefore improves the efficiency of the activity. These increasing returns are intrinsically dynamic. Kaldor reverts, on the aspect, to Young (1928) who attributes increasing returns to a large scale division of labour:

> "In addition, as Allyn Young emphasised, increasing returns is a 'macro-phenomenon'-just because so much of the economies of scale emerge as a result of increased differentiation, the emergence of new processes and new subsidiary industries, they cannot be 'disconnected adequately by observing the effect of variations in the size of an individual firm or of a particular industry'. (Kaldor (1966) p.9-10)"

Kaldor (1966) supports this two-fold explanation for the existence of increasing returns by re-interpreting the empirical evidences provided by Verdoorn (1949):

> "This [i.e. dynamic increasing returns], in my view, is the basic reason for the empirical relationship between the growth of productivity and the growth of production which has recently come to be known as the 'Verdoorn Law' [...]. It is dynamic rather than a static relationship... primarily because technical progress enters into it, and not just a reflection of the economies of large-scale production." (Kaldor (1966) p. 10)

Verdoorn's article initially aimed to find a method to forecast changes in labour productivity. As a matter of fact, his empirical investigations stressed the existence of a constant relationship between growth rates of labour productivity and of production for both the pre- and the post-World War I periods for a selected number of countries[1]. Kaldor (1966) himself estimated

[1] Depending on the period, Verdoorn's country sample included: Canada, Czechoslovakia, Estonia, Finland, Germany, Holland, Hungary, Italy, Japan, Norway, Poland, Switzerland, UK and USA.

the same relationship for the period 1953-1963 using a sample of 12 countries[2]. Further, he estimated a relation between employment and production growth rates[3]. Both the empirical relationships appeared to be significant and robust, and became known as the Kaldor Verdoorn Law.

The simplicity of the functional form of the Law represents a simple alternative to forecast productivity changes or quantify the magnitude of the increasing returns. It prevents the "abuse" of micro-level assumptions, either on the functional for of an aggregate production function or on the behaviour of economic agents. This might explain the relative popularity of the law. McCombie, Pugno, and Soro (2002) list about 80 major papers making use of the Kaldor-Verdoorn Law since Verdoorn (1949). Most of these contributions concern applied empirical analysis, making use of the law to measure increasing returns for specific countries, regions or sectors and/or to account for differences among the latter.

The empirical uses of the Law are confined to the identification and quantification of increasing returns. The Kaldor-Verdoorn Law in itself does not provide any explanation on the mechanisms underlying technical change or the existence of these increasing returns. The theoretical foundations brought to the Kaldor-Verdoorn Law by Kaldor (1966) remained verbal. Few formal micro-foundations of the law can be found in the literature: On the one hand, Verdoorn (1949) shows that the relationship at the core of the Law is compatible with various types of aggregated production functions without really providing micro-foundations. On the other hand, McCombie (2002) shows, while formalising Allyn Young's macro-level increasing returns, that under some conditions on the parameters of the industry level production function, a Kaldor-Verdoorn Law relationship can exists. He also argues that Romer (1986) and Romer (1990) provide another possible theoretical explanation for Young's theory. It has to be noted that in this case, increasing returns are assumed more than micro-founded. The lack of micro-foundations possibly resides in the fact that, first, in an equilibrium framework, increasing returns have to be assumed. Second, as stated by Kaldor (1972), dynamic increasing returns, as measured by the Law, can only be considered out of equilibrium, and necessary generates disequilibrium.

As argued in Llerena and Lorentz (2004a), the Schumpeterian/Evolutionary approach to technical change, should provide a formal framework to micro-

[2] Austria, Belgium, Canada, Denmark, France, West Germany, Italy, Japan, Netherlands, Norway, UK and USA.

[3] This relation is known as the 'Kaldor specification' of the Kaldor-Verdoorn Law. It is also known, for other reasons, as the 'Fabricant Law' (see Metcalfe, Foster, and Ramlogan, 2006)

found the Kaldor-Verdoorn Law. Within the Evolutionary framework, and following Schumpeter's precepts, technical change emerges at the micro-level and diffuses into the economy. These mechanisms are at the core of economic dynamics. Contrary to the NGT that assumes increasing returns to generate technical change, here, technical change generates the dynamics; increasing returns being its emergent property.

The paper aims to show that a Kaldor-Verdoorn Law can emerge from the dynamics generated by an Evolutionary model of technical change. In this sense, we aim to provide a micro-level explanation for the Law complementary to the ones verbally provided by Kaldor (1966).

The remainder of the paper is organised as follows: Section 2 describes the model we use for the numerical simulations whose results are presented and analysed in section 3. Section 4 concludes the paper.

2 An Evolutionary model of technical change

We develop here a simple model of firm level technical change in line with the Evolutionary literature. We refer along this paper, more specifically to the Neo-Schumpeterian branch of Evolutionary economics (see Witt, 2008). Developed around the seminal work of Nelson and Winter (1982) [4], this stream of literature proposes a formal representation of Schumpeter's thought: Technical change, as a key factor for economic dynamics, emerges unevenly and unpredictably from firms or entrepreneurs' behaviour. It then diffuses across the economy, disrupting the established economic equilibrium.

The evolutionary interpretation of Schumpeter's analysis relies on analogies with the formalisation to be found in evolutionary biology. Note that the analogy remains quite limited here. An economic system is assumed to be composed by one (or more) population of agents. The latter are defined by a set of heterogeneous characteristics. The characteristics of the population(s) are subject to mutations occurring unevenly among agents, generating and sustaining the heterogeneity within populations. The agents composing the population(s) are subject to selection mechanisms. The selection mechanism defines the level of performance of the agents and there survival. In the Neo-Schumpeterian literature, the selection mechanism is usually considered to represent (or to be represented by) market mechanisms.

Following the traditional modelling strategies to be found in this stream of the evolutionary literature, the model is organised as follows:

[4] See also Dosi, Freeman, Nelson, Silverberg, and Soete (1988), Kwasnicki (2001) and Silverberg and Verspagen (2005) for more recent developments in this stream of literature.

1. A population of I firms, indexed $i \in \{1; ...; I\}$, characterised by a homogenous product but heterogeneous production processes leading to differences in productivity, price and profitability.
2. The selection mechanism shares the total demand among firms, favouring the best performing firms. Selection acts here as a channel for the diffusion of the most efficient technologies, ruling out the least efficient firms from the market.
3. Firm's production process is subject to mutation through technological shocks linked to there R&D activity. We choose here to distinguish two phases in this process:
   (a) Exploration: Firms search for new production facilities, through innovation or imitation of existing production facilities. The outcome is uncertain and defines efficiency (in terms of productivity) of the resulting vintage of capital goods.
   (b) Exploitation of R&D outcome: This stage requires firms to invest in incorporating the outcome of research in the production process. This second stage is funded by firms' sales, and then directly dependent on the success of previous investments.

The firms are assumed to be bounded rational. They are not conscious of the selection mechanisms, and do not directly respond to it but revert to simple decision rules to set both their prices, their investment and technological adoption strategies. The aggregated dynamics as analysed in the third part of the paper is derived from the combination of these micro-elements. Note that we focus here on the supply side dynamics to put in light the technological mechanisms that can lead to the emergence of a Kaldor-Verdoorn Law. We therefore leave aside the mechanisms expanding aggregate demand, complement of the Kaldor-Verdoorn law in generating cumulative growth.[5]

This section is organised as follows: We first characterise the agents composing our population, then describe the selection process, and end presenting of the mutation mechanisms.

2.1 Defining the population: Firms characteristics.

Our model is structured around a population of firms $i \in [1; I]$. In the short run (i.e., at each time step t), a given firm i is represented by a produc-

[5]Models integrating such Evolutionary micro-dynamics in a complete cumulative growth framework can be found among others in Llerena and Lorentz (2004b), Lorentz (2006) or Lorentz (2008).

tion function characterised by constant returns to scale. Firms' production process uses labour as a unique production factor. Capital enters indirectly in the production function. The level of labour productivity depends on the accumulation of capital vintages. Investment in the different vintages of capital goods will increase labour productivity. The production function is represented as follows:

$$Y_{i,t} = A_{i,t-1} L_{i,t} \tag{1}$$

where $Y_{i,t}$ is the output of firm i at time t. $A_{i,t-1}$ represents labour productivity and $L_{i,t}$ the labour force employed in the production process. The output is constrained by the demand for the firms' products. The level of aggregate demand (D_t) is set exogenously [6] . Aggregate demand is then allocated to each firm according to the selection process setting firms' market shares ($z_{i,t}$). The level of production of each firm is computed as follows:

$$Y_{i,t} = z_{i,t} D_t \tag{2}$$

Firms set prices through a mark-up rule. This mark-up is applied to unit production costs. Formally, the pricing rule can be represented as follows:

$$p_{i,t} = (1 + \mu) \frac{w}{A_{i,t-1}} \tag{3}$$

where $p_{i,t}$ represents the price set by firm i at time t, μ the mark-up coefficient and w the nominal wage set exogenously. It should be noted that we assume here that the mark-up coefficients are fixed for each firms.

The firm's profit level is then computed as follows:

$$\Pi_{i,t} = p_{i,t} Y_{i,t} - w L_{i,t} = \mu \frac{w}{A_{i,t-1}} Y_{i,t} \tag{4}$$

In the model profits constitutes the only financial resource for firms' investments. In other words, all the decisions taken by the firms are constrained by their profits. Their ability to capture demand shares, due to their past performances therefore directly affects all their investment plans.

2.2 Defining firms' performance: The selection mechanisms.

The selection mechanism represents, in an evolutionary system, the core of its dynamics. It sorts the various components of a population, creating motion

[6]We assume here that the aggregate demand grows at a fixed rate δ, so that:

$$D_t = D_{t-1} (1 + \delta)$$

in the system, and allocating resources within the population. The selection process is usually considered by Evolutionary economics as a metaphor for the competition mechanisms. We choose here to use a replicator dynamics to model the selection mechanisms [7]. The replicator dynamics is usually considered as a formal representation of Fisher's principle of natural selection. This principle can be summarised as follows: The share of groups of individuals in a population is favoured by their relative fitness level with respect to the average. This average level depends itself on the shares of every groups, such that the selection mechanisms tend to favour the fittest components of a population. Formally the replicator equation defines the increase (decrease) in the share of a group of individuals as a function of the distance between the fitness level of the group and the average fitness level. The higher the distance, the higher the share grows.

We use this mechanism to model the competition among firms. The goods produced being homogeneous, the competition among firms is based on prices. The replicator dynamics defines firms' market shares as a function of firms price competitiveness ($E_{i,t}$). The lower the price ($p_{i,t}$), the higher price competitiveness: $E_{i,t} = \frac{1}{p_{i,t}}$

Formally, the replicator mechanism modifies firms market shares ($z_{i,t}$) as a function of price competitiveness ($E_{i,t}$) relative to the average competitiveness on the market ($\bar{E}_t$)[8]:

$$z_{i,t} = z_{i,t-1} \left(1 + \phi \left(\frac{E_{i,t}}{\bar{E}_t} - 1\right)\right) \tag{5}$$

The parameter ϕ measures sensitivity to changes in competitiveness. ϕ can be assimilated to the price elasticity of the selection mechanism; the closer ϕ to 0 the more rigid the selection with respect to price competitiveness.

Firms exit the market if their market share is below $\bar{z}$. The exiting firms are replaced by entrant firms whose characteristics are set equal to the market averages, with an entry market share equal to $\bar{z}$. This insures a constant number of firms on the market as required by the formalisation of the replicator equation above to insure market shares to sum to one and $z_{i,t} \in [0;1]$,

[7] A comprehensive view on the use of the replicator dynamics in Evolutionary economics can be found in Metcalfe (1998)

[8] $\bar{E}_t$ the average competitiveness on the market, given by:

$$\bar{E}_t = \sum_i z_{i,t-1} E_{i,t}$$

$\forall i \in I$. Moreover, the exit of an innovator (imitator) firm is compensated by the entry of another innovator (imitator) firm. The proportion of innovators in the population of firms also remains constant.[9]

2.3 Changes in Firms characteristics: The mutation mechanisms

The mutation mechanisms insure that the system remains in motion, counterbalancing the selection mechanisms. The selection dynamics requires some degrees of heterogeneity among the characteristics of the agents. Through time, selection limits the level of heterogeneity in the system. The mutation of agents' characteristics generates and sustains some degree of heterogeneity within the population.

In our model, mutation occurs at the level of the production processes of the firms through changes in labour productivity. The process of technical improvement can be divided into two distinct phases. Firms explore new technological possibilities, through local search (innovation) or by capturing external technological possibilities (imitation). This process leads to a production design (capital vintage) that can be exploited by firms in their production process. The second stage consists in incorporating this new capital vintage. The exploitation process is related to investment in capital goods and the exploration is related to investments in R&D. We assume that a priority is given to capital investments, and therefore the exploitation of already discovered technologies.

Labour productivity is deduced from the accumulation of capital goods through time. Each vintage of capital embodies a level of labour productivity ($a_{i,t}$). The more a firm invests in a vintage the more its level of embodied labour productivity affects the production process. At every time step labour productivity can be expressed by the following equation:

$$A_{i,t} = \frac{I_{i,t}}{\sum_{\tau=1}^{t} I_{i,\tau}} a_{i,t-1} + \frac{\sum_{\tau=1}^{t-1} I_{i,\tau}}{\sum_{\tau=1}^{t} I_{i,\tau}} A_{i,t-1} \tag{6}$$

where $a_{i,t-1}$ represents the labour productivity embodied in the capital good developed by i during period $t-1$. $I_{i,t}$ represents the level of investment in capital goods of the firm.

[9]Note that this assumption has no qualitative effect on the results, as long as there exist at least one innovator.

The level of investments in capital goods ($I_{i,t}$) corresponds to a share (ι_i) of firms' sales ($Y_{i,t}$). To simplify the model, we assume ι_i to be fixed. We exclude the possibility for investments to be adapted and used as a strategic variable, contrary to Silverberg and Verspagen (1994), Silverberg and Verspagen (1998) among others[10]. This assumption allows us to isolate the effect of ι_i on the Kaldor–Verdoorn Law.

Investments are financed by firms' own resources and cannot exceed the actual sales: $\iota_i \in [0;1]$. For the seek of simplicity we rule out the possibility for firms to revert to a financial sector to finance their investments. Firms are therefore constrained by their own profits to finance investments:

$$I_{i,t} = min\left\{\iota_i Y_{i,t}\ ;\ \Pi_{i,t}\right\}$$

The numerical values of the parameters used in the simulations insure that $\iota_i < \mu \frac{w}{A_{i,t-1}}$. The actual investments are always below the profit constraint, and can be expressed as follows[11]:

$$I_{i,t} = \iota_i Y_{i,t} \tag{7}$$

The resources available for investments depend on the firms' sales, a function of their competitiveness and therefore depend on their previous performances. The model does not only account for the endogenous construction of the production capacities of firms, through the accumulation of capital. This process is also constrained by the firms' past performances. The model includes to its Evolutionary micro-foundation an additional 'Austrian' flavour (Amendola and Gaffard, 1998).

The level of investments in R&D corresponds to a share ρ_i of their sales ($Y_{i,t}$). For the seek of simplicity, this share remains fixed, and cannot exceed firms' own resources: $\rho_i \in [0;1]$. R&D investments are constrained by the remaining available profits ($\Pi_{i,t}$) after capital investments ($I_{i,t}$). The R&D investments are used to hire workers ($R_{i,t}$) assigned to the R&D activity:

$$R_{i,t} = \frac{1}{w}\min\{\rho_i Y_{i,t}; \Pi_{i,t} - I_{i,t}\} \tag{8}$$

[10]In Llerena and Lorentz (2004a), we considered investment behaviours as driven by adaptive decision rules. With such rules however, surviving firms tend to apply fixed shares in the long run, while firms lagging too far behind do no manage to compensate their gap adapting their behaviour. The priority given to investments in capital then prevents any R&D investment.

[11]Investments are growing at the same rate as firms' output. The dynamics for capital accumulation follows then exactly the growth of output at the firm level.

We assume here that firms resources are either invested in capital or in R&D, so that $\rho_i = 1 - \iota_i$.

In direct line with Nelson and Winter (1982), we consider the outcome of the R&D activity as uncertain: First, the probability of success of R&D is an increasing function of the R&D intensity of the firm, as measured by $\frac{R_{i,t}}{Y_{i,t}}$. Second, if successful, the characteristics of the newly developed capital vintage is stochastic, resulting from either a process of 'local search' or imitation, depending on the nature of the firm.

The R&D process, followed by each firms, is represented by the algorithm that follows:

1. A first random draw decides of the success (or failure) of the R&D activity at time t. The probability of success of the R&D process increases with the number of workers hired for the research activity. This probability is null if the ratio is null and tends to one if the firm uses all its resources to hire R&D workers.

2. If R&D is successful, the prototype of a new capital vintage is developed. The new vintage at t is characterised by an embodied level of labour productivity ($a_{i,t}$):

$$a_{i,t} = a_{i,t-1} + \max\{\epsilon_{i,t}; 0\} \tag{9}$$

$$\epsilon_{i,c,t} \sim N(0; \sigma_{i,t}) \tag{10}$$

   The value of the standard deviation $\sigma_{i,t}$ depends on the nature of the firm: If the firm is an innovator, $\sigma_{i,t}$ is fixed; if the firm is an imitator, $\sigma_{i,t}$ is a function of firm's technological gap:

$$\sigma_{i,t} = \begin{cases} \sigma & \text{if the firm is an innovator} \\ \max\{\chi(\bar{a}_t - a_{i,t}); 0\} & \text{if the firm is an imitator} \end{cases} \tag{11}$$

   where $\bar{a}_t$ represents the average level of embodied productivity.[12]

In the case of innovators, the formalisation of R&D can be assimilate to Nelson and Winter (1982) concept of 'local search', as the stochastic process is centred on the previous level of productivity, and the potential improvements

[12] It is formally computed as:

$$\bar{a}_t = \sum_i z_{i,t} a_{i,t-1}$$

limited. For imitators, the R&D activity consists in filling the gap with the industry's average technological level.[13].

3 Simulation Results

Our aim with this paper is two fold: While accounting for the contribution of the micro-level mechanisms underlying the emergence and diffusion of technical change on dynamic increasing returns at the aggregate level, we propose an alternative micro-foundation to the Kaldor-Verdoorn Law in line with the Schumpeterian tradition.

In the above presented model, firms experience constant returns to scale in the short run. Moreover, firms, on average, experience no changes in their production capacities. Hence, if they appear, increasing returns are dynamic and result from the evolutionary micro-dynamics.

To bring this result into light, we decompose the micro-dynamics into three main phases:

1. The emergence of technological shocks. This phase corresponds to the arrival of new capital vintages. It occurs at the level of the innovative firms as an outcome the R&D process.

2. The adoption of the technological shocks. This phase corresponds to the introduction of the new capital vintages in the production process.

3. The diffusion of the shocks among the population. The diffusion phase occurs through two channels: First through imitation; second through the selection process that allocates more resources to the fittest firms.

The presentation and analysis of the simulation results is organised as follows: Section 3.1 presents the parameter values used and reports preliminary results on the effect of the various parameter settings on the productivity dynamics. Section 3.2 presents the results of the estimation of the Kaldor-Verdoorn Law using the data generated by the numerical simulations.

3.1 Parameterisation and Preliminary Analysis

The three phases described above are directly affected by the following set of parameters. The simulations are then conducted in order to stress the effect

[13]In Llerena and Lorentz (2004a), firms that can switch from imitative to innovative strategies (and reversely) depending on their technological gap. Fixing the R&D strategies of the firms, as in this paper, does not affect qualitatively the results of the simulations. On the other hand, this allows us to isolate the effect of the diffusion of technologies through imitation from the diffusion within the population through selection.

of these parameters.

ι_i is defined in the previous section as the share of firms' resources devoted to investments in capital goods. This parameter controls the speed of adoption of the technological shock, generated by the R&D activity and embodied into the capital vintages. Moreover, we assume through the simulations procedure that ρ_i, the share of resources devoted to finance the R&D activity, is set equal to $(1 - \iota_i)$. ι_i also controls the frequency of technological changes, affecting directly the probability of success of R&D.

The parameter is set so that: $\iota_i \in [0; 1]$. As ι_i tends to 1, the allocation of resources by firms favours the adoption of existing technologies. As ι_i tends to 0, the allocation of resources favours the development of new technologies. Note that for extreme values ($\iota_i = 0; 1$), no productivity gains are possible: On the one hand, if all the resources are devoted to the adoption of existing technologies, this prevents to finance the R&D necessary for further advances in production technologies. On the other, if $\iota_i = 0$, all the resources are used to develop capital vintages that are never adopted by the firm.

σ corresponds to the standard deviation of the stochastic process defining the outcome of the R&D process for the innovator. This parameter controls the amplitude of the technological shocks resulting from the R&D activity of the firms. As σ increases, the potential jump in embodied productivity increases. Simultaneously, as only the positive jumps are adopted by firms, increasing σ also increases the unevenness of the technological shocks. This parameter therefore affects both the amplitude of technical changes at the firm level and the degree of heterogeneity among firms.

χ measures the appropriability of technological spillovers by imitators. This parameter controls the diffusion of technological shocks resulting from innovative firms to the imitative firms. The larger $\chi \in [0; 1]$, the higher the spillovers, the faster the imitators reduce their technological gap.

ϕ defines the sensitiveness of the replicator dynamics. This parameter directly controls the strength of selection. Through market selection, the available resources (aggregate demand) is allocated to firms. These resources are then used to develop and adopt production technologies. Increasing ϕ, first, increases the uneven distribution of resources resulting from the productivity differences among firms. Second, it reinforces the uneven technological endowments of firms favouring the access to resources for the high productivity firms. Finally, ϕ controls the diffusion of the best technologies within the economy.

δ constrains the rate of growth of aggregate demand, constraining directly the level of the resources available for firms. As we focus, in this paper, on the technological mechanisms underlying the Kaldor-Verdoorn Law, we choose to assume a fixed rate of growth. δ therefore controls the overall amount of resources to be distributed to firms. For a given distribution of resources among firms, and a given allocation of resources between capital and R&D investments within firms, increasing δ should accelerate the adoption of technologies by firms.

The parameter also allows us to control the total level of output in our population of firms. This appear useful to avoid any problem of endogeneity in the explanatory variable while estimating the coefficients of the Kaldor-Verdoorn Law in the second part of the analysis.

The first two parameters, ι_i and σ, allow us to identify the effect linked to both the emergence and the adoption of technological shocks at the firm level. The parameters ϕ and χ allow to isolate the effect of the phase of diffusion of these shocks among the population and to the economy as a whole. The parameter δ should theoretically amplify these mechanisms. The detailed parameter values used for the simulations are reported in Table 1.

[Table 1 about here.]

The numerical simulations are conducted as follows: Every simulation run lasts 500 time steps. This is largely sufficient for the dynamics generated by the model to stabilise. No more significant changes in the dynamics are observed beyond 500 periods.

Each parameter settings are replicated 50 times. The analysis relies on the average values over 50 replications. By doing so we insure the robustness of the results we present.

We consider for each simulation a population of 20 firms. One half of the firms behave as innovators, the second half as imitators. This repartition remains fixed along the simulations. The size of the population is sufficient to reach significant and robust results and insure reasonable computation times. A larger population of firms would not qualitatively modify the results.

Finally, we apply the same values for the initial conditions to all firms for all simulation runs. Heterogeneity among firms emerges only from the dynamics described by the model. The differences in the results presented for the various settings are only due to the specific parameters under investigation.

Some preliminary simulation analysis focuses on the effect of the various parameters controlling for the three phases of technical changes on productivity dynamics. We choose to measure productivity growth rates over the 50, 100 and 250 first simulation steps.

We first consider the join effect of parameters ι and σ. These parameters allow to control both the allocation of resources between capital and R&D investments and the standard deviation in productivity gains generated through R&D.

[Figure 1 about here.]

[Figure 2 about here.]

Figures 1 and 2 present the effect on aggregate productivity growth rates of variations in the values of ι and σ, over selected time periods. Simulations show that both parameters have a clear effect on productivity dynamics: Higher values of ι tend to decrease productivity growth, while increasing σ increases productivity growth. On the one hand, the more resources are devoted to R&D, the higher productivity growth. This effect is amplified as σ increases. On the other hand, the higher the amplitude of technical change, the higher productivity growth. This effect is amplified for low values of ι (see figure 1). Similar patterns emerge for the various time periods considered. However, the amplitude of these effects seems to gradually vanish as time goes (see Figures 2).

The probability of success of R&D being directly correlated with the share of resources devoted to R&D, the lower ι, the higher the probability of occurrence of technical change. More frequent novel capital goods arising hence favours productivity growth. However, if ι is too low ($\iota = 0$), firms don't invest in capital to exploit these changes. Favouring, the emergence phase increases productivity growth but requires a minimum resources to be devoted to the adoption to be effective.

The parameter σ controls the potential productivity jumps between two capital vintages. Increasing the amplitude of the shock then mechanically favours the productivity growth rate. This effect is reinforced as the frequency of the shocks is increased with higher shares of resources devoted to R&D.

These two parameters affect directly the phase of emergence and adoption of the technological change. The higher the frequency of the technological shocks, the higher the productivity growth. Second, the higher the amplitude of the technological change induced by these shocks, the higher productivity growth rates. On the one hand, the more firms favour the emergence of new capital vintages, the higher the aggregate productivity growth. On the other, adoption constrains the actual effect of technical change on productivity growth. If no resources are devoted to the adoption of these vintages, the effect of technical change on productivity disappears.

The second set of simulations focuses on the effect of changes in χ and ϕ on the productivity dynamics. The parameter χ controls the level of spillover to be absorbed by the imitating firms. The higher χ, the larger the absorption of spillover by the firms, the more these firms reduce their productivity gap. This parameter favours the diffusion of the shocks through imitation. The parameter ϕ controls the sensitiveness of the selection process. It mechanically affects the aggregated productivity dynamics: A stronger selection mechanism favours the most competitive firms that faster gain market shares. These firms benefited from better technological shocks. The higher ϕ, the faster the technological shocks diffuses among the system.

[Figure 3 about here.]

[Figure 4 about here.]

Figure 3 presents the outcome of the changes in these parameter on the productivity growth rates over 50 simulation steps. In this case, strengthening the selection mechanism clearly affects the productivity dynamics. This result can be explained by the mechanisms described above. The higher ϕ, the stronger the selection mechanisms and the faster the diffusion of the technological shocks. Increasing χ, on the other hand, only slightly increases productivity. This effect even vanishes when considering productivity growth over 100 steps (Figure 4a). Figure 4b presents the productivity growth rates over 250 simulation steps. The results previously found have disappeared, except for very loose selection pressures. In the latter case, firms have time to imitate explaining the productivity growth picks for the high values of χ.

The influence of the strength of the selection mechanism is only transitory. This is due to the nature of the selection mechanisms. The replicator dynamics necessarily leads the system to a quasi-monopoly situation. The aggregate dynamics is only due to the productivity changes of the monopolistic firm, and the underlying Schumpeterian micro-dynamics stabilise. The factors favouring diffusion are then marginal. This result is directly linked to some assumptions made to keep the model simple: Including changes in technological trajectories or large scale division of labour, should prevent the Schumpeterian dynamics to stabilise.

We can briefly summarise the results found as follows:

- Factors favouring the emergence of technical shocks at the firm level seem to prevail all the other mechanisms.

- As selection mechanisms go all the effects tend to gradually disappear; technical change becoming less likely.

- In all cases, the effect of these parameters on productivity dynamics vanishes through time .

We might then infer that if increasing returns are to be found, these might be limited in time, and mainly influenced by the frequency and the amplitude of firm level shocks.

3.2 The Kaldor-Verdoorn Law as an emergent property

The second part of the simulation analysis aims to show that a Kaldor-Verdoorn Law can emerge from the dynamics generated by a simple Evolutionary model. We estimate the Verdoorn specification of the Law as follows:

$$\frac{A_t - A_0}{A_0} = \alpha + \lambda \frac{Y_t - Y_0}{Y_0}$$

A_t is the aggregate level of productivity as generated by the simulations, and Y_t measures the aggregate output. This equation is then estimated using the data generated by the simulation model.

The data set is built as follows: The aggregate output is defined by aggregate demand. Aggregate demand grows at an exogenous growth rate. We use exactly the same values of this growth rates for all the parameter settings. The data set for aggregate productivity is generated by the various replications of the simulation for different values of the parameters.

We estimate the Kaldor-Verdoorn Law for the various specifications of the parameters. We then analyse the effect of the changes in the values of these parameters on the estimated value of the Verdoorn coefficient, its significance level, and on the explanatory power of the estimates. The significance level is measured using Student t, and the explanatory power of the estimates using the corrected R^2.

We focus here on the estimates realised for the 50 steps growth rates. In all cases, estimates of the Verdoorn coefficient are significant (except for extreme values of the parameters). In other words a Kaldor-Verdoorn Law emerges regardless the parameter settings. In this respect our micro-founded model is able to generate significant dynamic increasing returns as an emergent property of the micro-dynamics. In this sense the model provides an alternative micro-foundations to the Kaldor-Verdoorn Law.

Note that as expected, the law disappears for larger time-span: it appears less frequently for the 100 step growth rates and do not appear for the 250 step growth rates. Technical change occurring more unevenly in these case, one cannot expect significant dynamic increasing returns to occur.

The first set of estimates focuses on the parameters controlling the emergence of the technological shocks. The results of these estimates are presented in Figures 5 to 7.

[Figure 5 about here.]

Figure 5 presents the estimated values of the Verdoorn coefficient for different settings of ι and σ. These parameters respectively influence the frequency and the amplitude of technological shocks. As seen in the previous sub-section, these parameters directly affect productivity dynamics. They should therefore positively affect the level of the increasing returns. This is confirmed by the estimated values of the Verdoorn coefficient. First, increasing the value of σ significantly increases the value of the coefficient. Second, increasing the values ι reduces the value of the coefficient. The parameters favouring the emergence of the shock positively affect the level of the increasing returns.

[Figure 6 about here.]

Figure 6 presents both the standard deviation and the value of Student t statistics for the estimated coefficients. We focus on this statistic to measure the effect of the changes in the parameters on the significance level of the estimated coefficient. This figure clearly shows a decrease in the Student's t when increasing both ι and σ. This decrease is such that for high values of σ only few of the estimated coefficients are significantly different from zero. Hence, if an increase in the amplitude of the technological shocks increases the value of the Verdoorn coefficient, it tends to be less significant. This tendency is also confirmed by the Figure 7 that presents the corrected R^2 of the estimations. Increases in σ also correspond to drastic decreases in this statistic. In other words, increasing the amplitude of technological changes increases the degree of increasing returns but these are less and less significant and show lower explanatory power. This result is due to the stochastic nature of the technological change: Increasing σ enlarges the potential productivity gains at the cost of more uneven gains. These increases in the variability of the technological shocks explains the loss in terms of significance.

[Figure 7 about here.]

Result 1 *The higher the amplitude of innovation (σ), the higher the Verdoorn coefficient, therefore, the higher the increasing returns. Simultaneously the estimated coefficients loose of their significance and the Law its explanatory power. The higher the investments in R&D (lower ι), the higher increasing returns. Higher investments in R&D levels preserves the significance of the Law for high values of σ.*

The second set of estimations concerns the parameters affecting the diffusion of the technological shocks on the system. We focus the analysis on the effect of changes in the strength of the selection mechanisms (ϕ) and the appropriability of the spillovers (χ) on the estimated value of the Verdoorn coefficient. Figures 8 to 10 respectively present the estimated value of the Verdoorn coefficient, its Student t statistic and the corrected R^2 for the various specifications of the two parameters.

[Figure 8 about here.]

The previous section highlighted the significant but transitory effect of strengthening the selection mechanisms on productivity dynamics. As shown by Figure 8, increasing the sensitiveness of the selection process has a positive effect on the estimated values of the coefficient of the Law.

[Figure 9 about here.]

[Figure 10 about here.]

A strong selection process mechanically increases the aggregate productivity dynamics, giving more weight to the most dynamic firms. This effect is then disclosed, through the estimates of the Kaldor-Verdoorn Law, into a higher the level of increasing returns. A more striking results also comes out from the estimations of the Law: Changes in the parameter χ positively and significantly affect the estimated value of the coefficient. Hence, favouring imitation favours increasing returns.

Figure 9 presents the value of the Student t statistics for the estimated Verdoorn coefficients. These measure the level of significance of the coefficient. Except for the highest values of ϕ, all the estimated coefficient are significantly higher then zero. Note, however, that the value of the statistic decreases as χ increases. A stronger selection mechanism limits the number of firms able to generate technical change. The aggregate productivity growth is then more sensitive to the uneven nature of the shock. The outcome of the estimations is then less significant.

Figure 10, presenting the values of the corrected R^2, corroborates this result. As the selection mechanism gets stronger, the value of this statistic decreases. The Law therefore looses its explanatory power. The changes in the parameter χ slightly positively affect the two statistics. More generally the effect of these parameters on both the estimates of λ and the statistics considered remains limited. The range of the changes evolved is largely less significant than for the first set of parameters.

Result 2 *Enforcing the selection mechanisms significantly affects the Kaldor-Verdoorn Law. It favours, as imitation mechanisms, the diffusion of technological shocks this increase the value of the estimated coefficient. A stronger selection mechanisms limits the number of active firms. This limits the frequency of the technical change and therefore reduces the significance and explanatory power of the Law.*

The estimations realised over this artificial data-set lead to the following results (as summarised in table 2): On the one hand, the frequency and the persistence of technological shocks favours both value and significance of the dynamic increasing returns, strengthening the Kaldor-Verdoorn Law. On the other hand, the factors favouring the amplitude and the diffusion of the shocks positively affect the level of increasing returns, although to the detriment of the significance of the coefficient, weakening the Kaldor-Verdoorn Law.

[Table 2 about here.]

In other words, dynamic increasing returns require frequent technical shocks to occur. The more frequent these shock the higher the returns. However, the amplitude of these shocks might prevent these returns to be significant. Too large productivity jumps at the firm levels are detrimental for the aggregate level increasing returns. Similarly when market pressures are too important. These results seem in line with recent empirical estimations of the Kaldor-Verdoorn Law (see among others Knell, 2004; Lorentz, 2005): Sectors experiencing higher (and/or significant) dynamic increasing returns are mainly activities (both in manufacturing and services) characterised by established technologies and/or constant technical improvements. For sectors experiencing important technical changes or technological breakthrough (as ICTs related industries and services or aircraft and space craft industries in Lorentz (2005)), the Kaldor-Verdoorn Law is more rarely observed.

These results somehow relativise the predominant role of technological breakthrough as explaining economic growth as usually put forward in the Schumpeterian literature. More frequent incremental improvements along a given technological trajectory seem more favourable to increasing returns then a rugged technological trajectory. These breakthrough remain important in favouring the emergence of new trajectories which cannot be accounted for in our model.

These results also show that capital accumulation cannot in itself account for the existence of increasing returns. The accumulation of capital affects increasing returns only as a mean of transportation for novelty. This brings us back to the very essence of the "technical progress function" as put forward by Kaldor (1957).

4 Concluding Remarks

This paper develops a simple micro-founded model of technological change inspired by the evolutionary literature. We have aimed to identify some sources of dynamic increasing returns. In this respect, we analyse the effects of changes in various micro-characteristics of the model on the productivity dynamics. This analysis highlights the importance of the frequency and amplitude of the technological shocks in shaping the aggregate productivity dynamics. Second, the simulation exhibits that the resources devoted to the adoption of these shocks only transitorily affect these dynamics. Similarly, the factors favouring the diffusion of these shocks, and particularly, the selection mechanism, have a significant but transitory effect on productivity dynamics.

We then estimate the Kaldor-Verdoorn Law using the data generated by the simulations for the various specifications of the parameters. The Law is verified in most of the cases. Moreover the estimation showed that some of the micro-characteristics affect the value and significance level of the Verdoorn coefficient. On the one hand, increasing the amplitude of the shocks and the strength of the selection mechanisms increases the values but decreases the significance of the coefficient. These losses in significance are respectively due to increase in the unevenness of the shocks, in the first case, and a reduction of the frequency of the shocks, in the second one. On the other hand, augmenting the resources devoted to R&D increases the frequency of the shocks, affecting positively the value and significance of the shocks. The results from these estimations therefore show the limited impact breakthrough technical change can possibly have on the aggregate productivity dynamics.

We showed through the paper that Schumpeterian micro-dynamics constitutes an alternative explanation for the emergence of a Kaldor-Verdoorn Law. The various components of these micro-dynamics have a rather complex effect on the Law: Reinforcing the increasing returns and in some cases, simultaneously limiting their significance due to higher unevenness in technological changes. These results, however, are only transitory: As the Schumpeterian dynamics stabilises, the Law is no longer observable. This can be explained by some of the assumptions made designing the model: Technological changes are only 'local'; there is no changes of trajectory or emergence of new paradigms that would prevent the stabilisation of the Schumpeterian dynamics. Further, the model does not account for large scale division of labour, either through the emergence of new sectors or the externalisation of parts of firms activities. Such types of considerations require to rethink the formalisation of the production process beyond what the Evolutionary literature proposes. This clearly opens the door for future developments of the present work.

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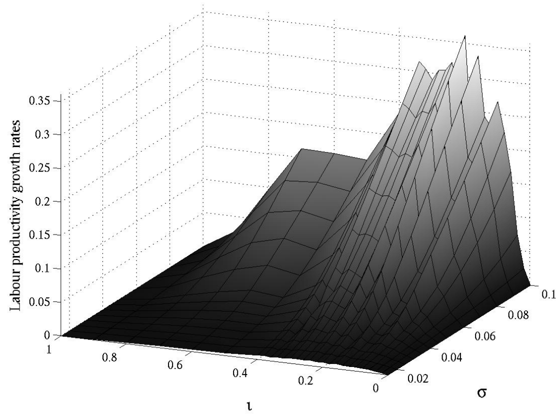
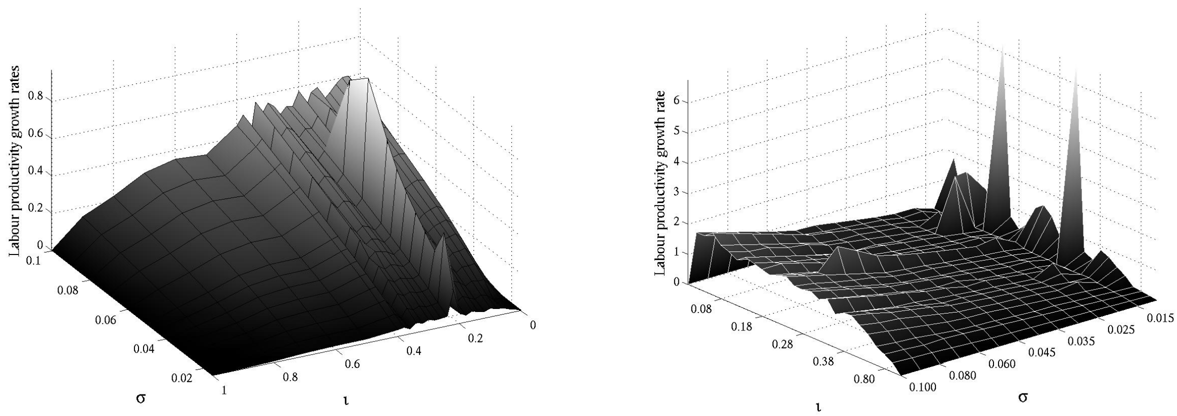


Figure 1: Labour productivity growth rates (50 steps), ι vs. σ



(**a**) Productivity growth rates (100 steps) (**b**) Productivity growth rates (250 steps)

Figure 2: Labour productivity growth rates, ι vs. σ

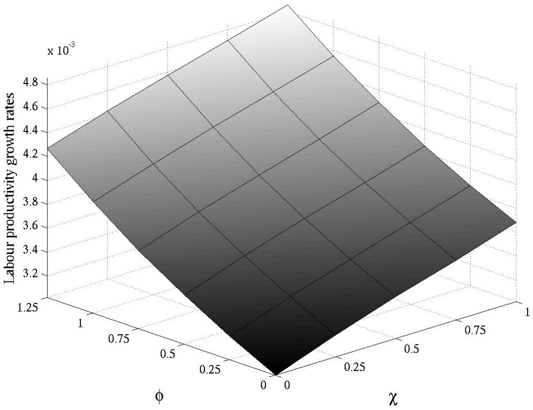
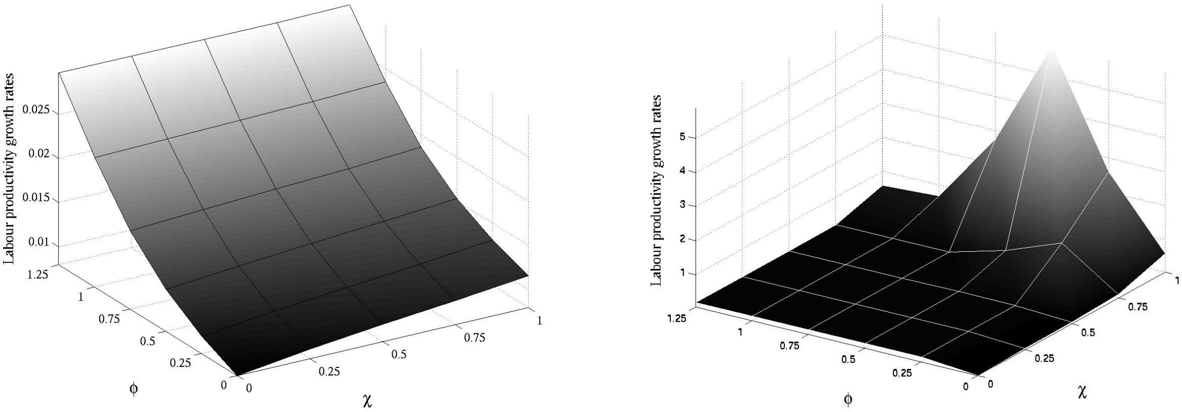


Figure 3: Labour productivity growth rates (50 steps), ϕ vs. χ



(**a**) Productivity growth rates (100 steps)

(**b**) Productivity growth rates (250 steps)

Figure 4: Labour productivity growth rates, ϕ vs. χ

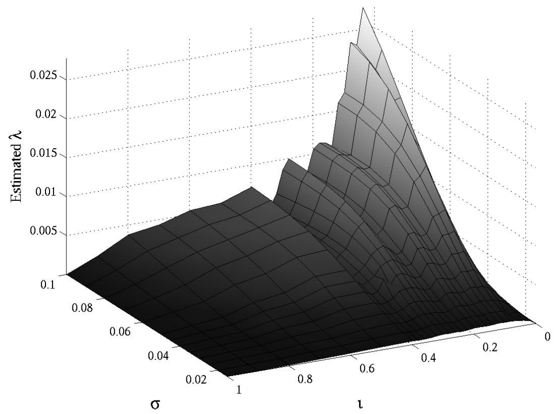
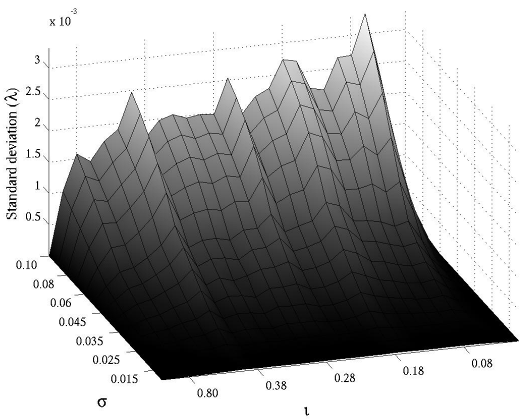
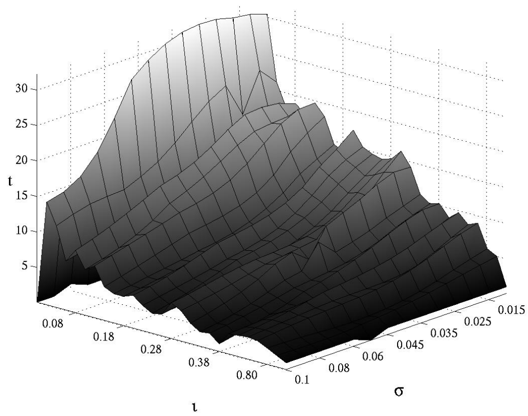


Figure 5: Estimated Verdoorn coefficient (ι vs. σ)



(**a**) Standard deviation



(**b**) Student's t

Figure 6: Statistics for the Verdoorn coefficient, ι vs. σ

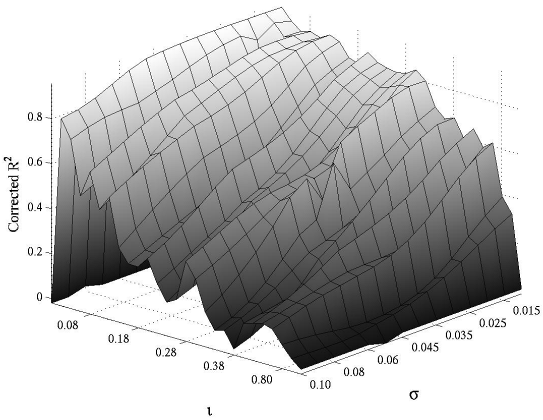


Figure 7: Corrected R^2 for the estimated Kaldor-Verdoorn Law (ι vs. σ)

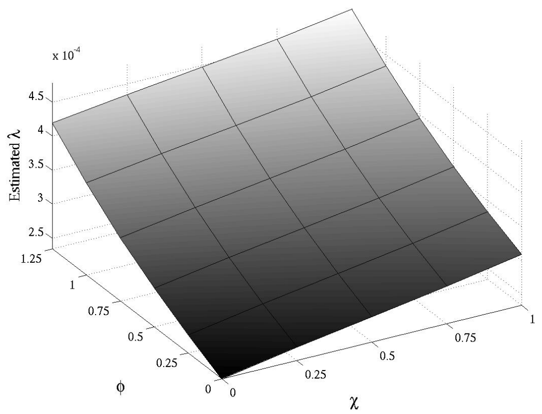
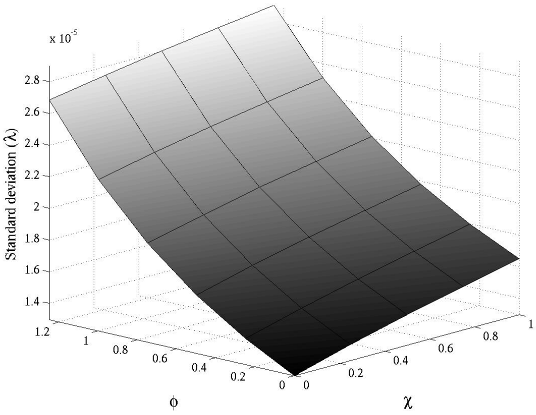
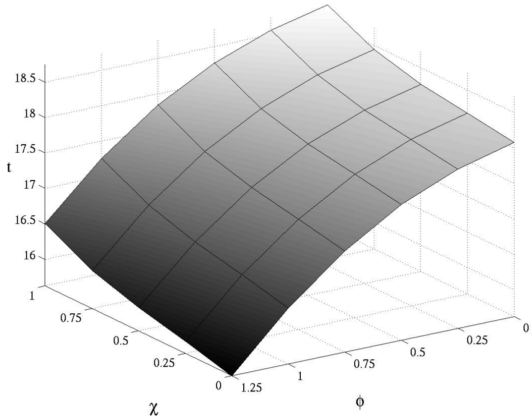


Figure 8: Estimated Verdoorn coefficient (ϕ vs. χ)



(**a**) Standard deviation



(**b**) Student's t

Figure 9: Statistics for the Verdoorn coefficient, ϕ vs. χ

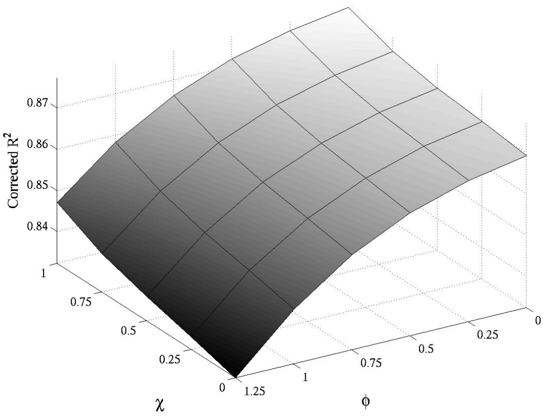


Figure 10: Corrected R^2 for the estimated Kaldor-Verdoorn Law (ϕ vs. χ)

Table 1: Parameters settings (default values in bold)

ι_i	0	0,02	0,04	0,06	0,08	0,1	0,12	0,14	0,16	0,18
	0,2	0,22	0,24	0,26	0,28	0,30	0,32	0,34	0,36	0,38
	0,40	0,50	0,60	0,70	0,80	0,90	1,00	-	-	-
σ	0,01	0,015	0,02	0,025	0,03	0,035	0,04	0,045	**0,05**	0,06
	0.07	0,08	0,09	0,1	-	-	-	-	-	-
δ	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009	**0,01**
	0,011	0,012	0,013	0,0,14	0,015	0,016	0,017	0,018	0,019	0,02
	0,021	0,22	0,023	0,024	0,025	0,026	0,027	0,028	0,029	0,03
	0,031	0,032	0,033	0.034	0.035	0.036	0.037	0.038	0.039	0.04
	0.041	0.042	0.043	0.044	0.045	0.046	0.047	0.048	0.049	0.05
ϕ	0	0,25	0,5	0,75	**1**	1,25	-	-	-	-
χ	0	0,25	0,5	**0,75**	1	-	-	-	-	-
μ	1	-	-	-	-	-	-	-	-	-
w	10	-	-	-	-	-	-	-	-	-
$\bar{z}$	0,0001	-	-	-	-	-	-	-	-	-
D_0	10	-	-	-	-	-	-	-	-	-
$z_{i,0}$	0,05	-	-	-	-	-	-	-	-	-
$A_{i,0}$	1	-	-	-	-	-	-	-	-	-
$a_{i,0}$	1	-	-	-	-	-	-	-	-	-

Table 2: Main Simulation Results

	$\nearrow \iota$	$\nearrow \sigma$	$\nearrow \phi$	$\nearrow \chi$
Verdoorn coefficient	$-$	$+$	$+$	$+$
Standard deviation	$-$	$+$	$+$	$+$
Student t	$-$	$-$	$-$	$+$
Corrected R^2	$-$	$-$	$-$	$+$